THERMAL CONDUCTION AND THE STABILITY OF HOT ACCRETION FLOWS

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ABSTRACT

Recently, Medvedev & Narayan (2001) discovered a new type of accretion flow, a hot settling flow around a rapidly rotating neutron star. The flow is cooling-dominated and energetically similar to the Shapiro, Lightman, & Eardley (1976, SLE) solution. Since the SLE solution is known to be thermally unstable, one might suspect that the new solution would also be unstable. However, due to the very high temperature of the accreting gas, thermal conduction is very strong and could suppress the thermal instability. We analyze the role of thermal conduction in both the hot settling flow and the SLE solution. In the hot settling flow collisions are very rare. Therefore, thermal transport occurs via free streaming of electrons along tangled magnetic field lines. We find that conduction is strong enough to make the flow marginally stable. In contrast, in the cooler SLE solution, conduction is via collisional, Spitzer-type transport. In this case, conduction is weaker, and we find that the SLE solution is thermally unstable even in the presence of conduction.

Subject headings: accretion, accretion disks — instabilities — conduction — magnetic fields — stars: neutron — black hole physics

1. INTRODUCTION

Accretion flows around compact objects frequently radiate significant levels of hard X-rays, indicating the presence of hot optically-thin gas in these systems. A number of hot accretion solutions have been discussed in the literature, e.g., the Shapiro, Lightman, & Eardley (1976) (SLE) solution, the advection-dominated accretion flow (ADAF) (Narayan & Yi 1994, 1995a,b; Abramowicz et al. 1995), and the convection-dominated accretion flow (CDAF) (Narayan et al. 2000; Quataert & Gruzinov 2000). These solutions are relevant for accretion onto a black hole. Recently, Medvedev & Narayan (2001) discovered a solution that corresponds to accretion onto a neutron star (NS). We refer to this solution as a "hot settling flow," since the gas is hot (the temperature is nearly virial) and it "settles" onto the NS (the ratio of the radial velocity to the free-fall velocity decreases with decreasing radius).

Not all hot accretion flows are stable. The cooling-dominated SLE solution has been shown to be thermally unstable (Piran 1978; Wandel & Liang 1991; Narayan & Yi 1995b) and, hence, unlikely to exist in nature. More generally, it has been shown that any accretion flow in which heating balances cooling is thermally unstable if the cooling is due to bremsstrahlung emission (Shakura & Sunyaev 1976; Piran 1978). The ADAF solution, on the other hand, is known to be thermally stable (Narayan & Yi 1995b; Kato et al. 1996, 1997). In this solution, cooling is weak (ideally zero), and so the thermal energy of the flow is not radiated but is advected with the gas (hence the name). The CDAF is also believed to be stable, since in this flow again the thermal energy is advected by convective eddies and is either carried into the black hole or is radiated near the outer boundary of the flow (Ball, Narayan & Quataert 2001).

The thermal stability of the Medvedev & Narayan (2001) hot settling flow has not been addressed so far. Energetically, this flow is very similar to the SLE solution, since the heat energy produced by viscous dissipation is radiated locally via bremsstrahlung. One might therefore expect the flow to be thermally unstable. However, this is not necessarily the case, as we show in this paper.

The temperature of the gas in all these accretion solutions is very high, reaching almost the virial temperature in several cases. Thermal conduction is thus likely to be enormous and could have a significant effect. The question that we try to answer in this paper is: How does conduction affect the thermal stability of the hot settling flow and the SLE solution?

In the hot settling flow, the mean-free-path is much larger than the flow scale (the radius); therefore, the Spitzer theory (Spitzer 1962) of thermal conduction cannot be applied. Instead we need to apply the transport theory for collisionless plasmas with tangled (but dynamically unimportant) magnetic fields (Rechester & Rosenbluth 1978; Chandran & Cowley 1998; Malyshkin & Kulsrud 2001; Malyshkin 2000). In the cooler SLE solution, collisions are more frequent; hence it is appropriate to use collisional transport theory modified for the presence of tangled fields.

The paper is organized as follows. We begin with a formal discussion of the thermal instability in §2, and discuss the stabilizing effect of thermal conduction in §3. In §4 and §5 we study the effect of thermal conduction on the hot settling flow and the SLE flow, respectively, and we conclude with a discussion in §6.

2. THERMAL INSTABILITY IN AN ACCRETION FLOW

The physics of the thermal instability is simple (Field 1965). Suppose a system is in thermal equilibrium, so that

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the rates of heating and cooling per unit volume are equal: $Q^+ = Q^-$. For simplicity let us take the heating and cooling rates to be functions of only the local temperature: $Q^+ \propto T^\alpha$, $Q^- \propto T^\beta$ ($\alpha,\ \beta > 0$ for concreteness).

Suppose, with increasing temperature, the cooling rate rises faster than the heating rate, i.e., $\beta > \alpha$. Then a local perturbation which causes a small increase in the temperature will result in a net cooling of the gas: $Q^- > Q^+$. This will cause the temperature to return to its equilibrium value, which means that the gas will be thermally stable. (It is easily seen that this is true also for a small decrease in the temperature.) On the other hand, if $\alpha > \beta$, the gas is thermally unstable. For instance, if the temperature decreases slightly, cooling becomes stronger than heating and the system deviates from its equilibrium in a run-away manner.

To study the thermal stability of an accretion flow, we need to include additional physics, namely the effects of shear and rotation. The shearing sheet approximation (Goldreich & Lynden-Bell 1965; Goldreich & Tremaine 1978) is a convenient way of introducing the relevant physics without unnecessary technical complications. This approximation is quite accurate for perturbations on length scales much smaller than the local radius.

Conventionally, the shearing sheet coordinates are Cartesian with $x,\ y,\ z$ corresponding to the radial, azimuthal, and vertical directions, respectively. These coordinates are appropriate for describing the motion of a parcel of gas whose geometrical size is small compared to the local radius, R, of the flow (i.e., $x,\ y,\ z \ll R$), so that the effects of geometry and curvature are insignificant. It is convenient to compare the wave-vector k of a perturbation with $1/R$ and the frequency of a mode with the local Keplerian frequency $\Omega_K = \sqrt{GM/R^3}$, where M is the mass of the central object. The shearing sheet approximation is accurate for "local" small-scale perturbations with $kR \gg 1$. Perturbations with $kR \sim 1$ are global; their properties may be understood only through a global stability analysis.

We consider a shearing gas flow with unperturbed velocity given by

$$\mathbf{V}_0(x) = 2A\,x\,\hat{y}, \tag{1}$$

where $2A = d\mathbf{V}_0/dx$ is the shear frequency and "hat" denotes a unit vector. Note that we have neglected the radial velocity in the equilibrium flow since this component of the velocity is significantly smaller than the azimuthal velocity in both the hot settling flow and the SLE solution. To include the effect of rotation we assume that there is a Coriolis acceleration, described by an angular rotation frequency $\mathbf{\Omega} = \Omega\,\hat{z}$. The vorticity and epicyclic frequency are then given by

$$2B = 2A + 2\Omega, \qquad \kappa_{\rm epi} = 2(\Omega B)^{1/2}. \tag{2}$$

Both the hot settling flow and the SLE solution satisfy the Keplerian scaling, $\Omega \propto R^{-3/2}$. Therefore, for both solutions we have $2A = -(3/2)\Omega$, $2B = \Omega/2$ and $\kappa_{\rm epi} = \Omega$.

We assume that perturbations in the flow have structure only in the x direction, and we ignore motions in the z direction. We write the perturbations (represented by primes) in the velocity, density and sound speed as

$$\mathbf{V}'(x,t) = u(x,t)\,\hat{x} + v(x,t)\,\hat{y},$$

$$\rho'(x,t) = \rho_0\sigma(x,t),$$
$$c_s^{2\prime}(x,t) = a^2(x,t),$$

where ρ_0 and c_s^2 are the equilibrium values of the density and the square of the sound speed. Note that we define c_s to be the isothermal sound speed, so that the pressure is written as $p = \rho c_s^2$. By considering perturbations of the basic hydrodynamic equations, namely the continuity, radial momentum, azimuthal momentum and entropy equations, we obtain the following four equations,

$$\frac{\partial\sigma}{\partial t} + \frac{\partial u}{\partial x} = 0, \tag{3a}$$

$$\frac{\partial u}{\partial t} - 2\Omega v + c_s^2\frac{\partial\sigma}{\partial x} + \frac{\partial a^2}{\partial x} = 0, \tag{3b}$$

$$\frac{\partial v}{\partial t} + 2Bu = 0, \tag{3c}$$

$$\frac{\rho_0}{\gamma-1}\frac{\partial a^2}{\partial t} - \rho_0 c_s^2\frac{\partial\sigma}{\partial t} = \left(Q^+ + Q^-\right)', \tag{3d}$$

where we have used $d/dt = \partial/\partial t + V_{0x}\partial/\partial x \simeq \partial/\partial t$ since the inflow velocity V_{0x} is set to zero in our approximation. Note that gravity does not enter the perturbed equations (it of course enters the unperturbed equation, where in equilibrium it cancels the centrifugal acceleration). In the shearing sheet approximation, the gravitational acceleration is assumed to be independent of x; therefore, it does not contribute to the perturbed equations. For simplicity, we have neglected viscosity in the azimuthal momentum equation, though we do include viscous dissipation in the energy equation through the terms Q^+ and $Q^{+\prime}$.

For the heating and cooling rates, we make use of "realistic" expressions that represent the physics of viscous accretion flows. Thus we write

$$Q^+ = \alpha\frac{\rho c_s^2}{\Omega_K}\left(\frac{dV_{0y}}{dx}\right)^2 = 4\alpha A^2\frac{\rho c_s^2}{\Omega_K}, \qquad Q^- = -\mathcal{C}\rho^2\left(c_s^2\right)^n, \tag{4}$$

where $\alpha \sim 0.1$ is the standard Shakura-Sunyaev viscosity parameter, V_{0y} is the y-component of the unperturbed velocity, and $\mathcal{C}$ is a constant. We leave the index n in the cooling function unspecified for now, but we note that $n = 1/2$ corresponds to non-relativistic free-free (bremsstrahlung) cooling. In equilibrium, we have $Q_0^+ + Q_0^- = 0$, and for the perturbations we find

$$(Q^+ + Q^-)' = -4A^2\alpha\,\frac{\rho_0 c_s^2}{\Omega_K}\left(\sigma + (n-1)\,\frac{a^2}{c_s^2} - \frac{1}{A}\frac{dv}{dx}\right).$$

We assume that the perturbations in equations (3) are of the form $\exp(-i\omega t + ikx)$. Substituting in the above equations and solving, we obtain the following dispersion relation:

$$\omega\left[\frac{\omega}{\gamma-1} + \frac{i(n-1)}{\tau_{\rm cool}}\right]\left(\omega^2 - \kappa_{\rm epi}^2 - k^2c_s^2\right) - \omega\left[\omega + \frac{i(2B/A-1)}{\tau_{\rm cool}}\right]k^2c_s^2 = 0, \tag{5}$$

where

$$\tau_{\rm cool} = \left(\frac{\rho_0 c_s^2}{Q_0^\mp}\right) = \frac{\Omega_K}{4A^2\alpha} = \frac{4}{9\alpha s^2}\,\Omega_K^{-1} \tag{6}$$

is the cooling (heating) time of the gas and $s = \Omega/\Omega_K$ is the dimensionless angular velocity of the gas.

The dispersion relation (5) corresponds to purely radial perturbations. The same relation can be used also for perturbations in the vertical direction, except that we must set $\kappa_{\rm epi} = 0$. Perturbations in the azimuthal direction are more complicated. Because of the shear, a non-axisymmetric wave packet is distorted as a function of time, and must be analyzed by special techniques which are beyond the scope of this paper (see, e.g., T77,GT78).

Equation (5) is a fourth-order polynomial and has four roots corresponding to four modes. A flow is unstable if any of the four modes grows with time, i.e. if the corresponding root has Im $\omega > 0$. One of the roots of the dispersion relation is always $\omega = 0$. This root corresponds to the viscous mode, which in the present case is particularly simple because we neglected viscosity in the momentum equation. It is easy to show that if we introduce viscosity into the momentum equation the viscous mode would become stable, i.e., we will obtain Im $\omega < 0$. We do not consider the viscous mode further in this paper.

The physics of the remaining three modes may be understood by considering equation (5) in various limits. Consider first the limit $k \to 0$. In this limit, two of the roots are given by $\omega = \pm\kappa_{\rm epi}$, corresponding to simple epicyclic oscillations. In the opposite limit $k \to \infty$, the same roots are given by $\omega = \pm\gamma^{1/2}c_s k$, which shows that they correspond to sound waves. In the absence of heating and cooling (i.e. $\tau_{\rm cool} \to \infty$), we can obtain an exact solution for these roots which is valid for all k:

$$\omega^2 = \kappa_{\rm epi}^2 + \gamma c_s^2 k^2. \tag{7}$$

This is the standard dispersion relation for sound waves in a differentially rotating flow. The presence of γ is because the relevant sound speed is the adiabatic sound speed, $\gamma^{1/2}c_s$ (recall that c_s is defined to be the isothermal sound speed).

The final root of the dispersion relation (5) corresponds to the thermal mode. In the limit $k \to 0$, we obtain

$$\omega = i\,(\gamma - 1)\,\frac{(1-n)}{\tau_{\rm cool}}. \tag{8a}$$

We see that the mode is stable (for $\gamma > 1$) if $n > 1$ and unstable if $n < 1$. In the opposite limit $k \to \infty$, we find

$$\omega = i\,\frac{(\gamma-1)}{\gamma}\,\frac{(2-n-2B/A)}{\tau_{\rm cool}}, \tag{8b}$$

Now, the mode is stable if $n > 2(1 - B/A)$, i.e. $n > 8/3$ for our problem, and unstable otherwise. Note that an accretion flow that is cooled by free-free emission ($n = 1/2$) is unstable in both limits.

The dashed lines in Fig. 1 show the real and imaginary parts of the various roots of the dispersion relation (5) as functions of k for a realistic set of parameters. We have chosen $\kappa_{\rm epi}^2 = 0.5\Omega_K^2$ which corresponds to a spin parameter[2] $s = \sqrt{0.5}$, and we have set the free-free cooling time to be $\tau_{\rm cool,ff} = 10/\Omega_K$ which corresponds to $\alpha \simeq 0.1$ (see equation [6]). Since we are interested in hot accretion flows with nearly virial temperature, we expect the sound speed to be comparable to the free-fall velocity; therefore, we have set $c_s = \Omega_K R$. Other parameters are $\gamma = 5/3$ and $n = 1/2$.

In Fig. 1 we do not show the trivial viscous root $\omega = 0$. We label the two acoustic modes (propagating radially in opposite directions) as 1 and 2 and we label the thermal mode as 3. The frequencies of the acoustic modes are modified from the analytic solution given in equation (7), which was derived by neglecting cooling. The real parts of the roots are slightly perturbed, and both roots pick up an imaginary part. However, the imaginary parts are negative, which means that these modes are stable in the presence of cooling. The thermal mode, however, is unstable (Im $\omega > 0$) for all k (Fig. 1), as expected from the asymptotic analysis presented earlier. We focus on this mode in the rest of the paper.

3. STABILIZING EFFECT OF THERMAL CONDUCTION

It is easy to see that thermal conduction will tend to reduce the thermal instability. An unstable thermal mode of wave-vector k consists of a growing temperature perturbation of wave-length $2\pi/k$. Thermal conduction tends to smooth out this temperature perturbation through heat diffusion. If the rate at which the temperature perturbation grows is smaller than the rate at which it is smoothed out by conduction, then the instability will be suppressed and the mode will be stable. Otherwise, the mode will continue to grow, but at a somewhat reduced rate.

The rate at which fluctuations are smoothed out by conduction depends on the spatial scale of the perturbation. The smaller the scale (i.e. the larger the value of k), the faster the conduction, and the greater the stabilizing effect. Thus, we expect conduction to stabilize thermal modes with k greater than some critical $k_{\rm crit}$. Our task in this section and succeeding sections is to estimate $k_{\rm crit}$ through a quantitative analysis. If we find that $k_{\rm crit}R \gg 1$, then we conclude that the flow is thermally unstable. On the other hand, if we find that $k_{\rm crit}R \lesssim 1$, we may reasonably claim that the flow is thermally stable. Technically, for $k \sim 1/R$, we need to carry out a global analysis rather than the local analysis presented in this paper, but this is beyond the scope of the present paper.

Let us write the heat flux q due to thermal conduction as

$$q_{\rm cond} = -\kappa\nabla T, \tag{9}$$

where κ is the thermal conductivity coefficient. Thermal conductivity in a dense, fully ionized gas is given by the Spitzer (1962) formula,

$$\kappa_{\rm Sp} \approx 1.3nk_B v_T\lambda \simeq 6.2\times10^{-7}T_e^{5/2}\ {\rm erg/(s\ K\ cm)}. \tag{10}$$

Here $v_T = (k_B T_e/m_e)^{1/2}$ is the electron thermal speed, T_e is the electron temperature ($T_e = T$ for a one-temperature plasma), k_B is the Boltzmann constant, and

$$\lambda \simeq 10^4 T_e^2/n\ {\rm cm} \tag{11}$$

is the electron mean free path. Note that λ is independent of the mass of the particle.

[2]The value of $s \sim 0.7$ was chosen such as to illustrate the effect of rotation which is proportional to $\kappa_{\rm epi}^2 \propto s^2$. This value of s is somewhat higher than a typical spin of rotating neutron stars, which is around $s \sim 0.1$. For such s the effect of rotation is negligible.

In the collisionless regime, i.e., when the mean free path of an electron becomes comparable to or larger than the temperature gradient scale $\lambda \gtrsim T_e/|\nabla T_e|$, equation (9) for the heat flux is no longer valid. For an unmagnetized plasma, the heat flux takes the following saturated form (Cowie & McKee 1977),

$$q_{\rm sat} \simeq -C\rho c_s^3 \,{\rm sgn}(\nabla T), \tag{12}$$

where $C \sim 5$ is a numerical constant whose exact value depends on the particle distribution function. This result is not relevant for our problem since our plasma is magnetized.

For a collisionless magnetized plasma, thermal conduction is anisotropic. Electrons stream freely along the field lines, and the parallel heat flux remains the same as for the unmagnetized case described above. However, the transverse heat flux is greatly reduced because electrons are tied to the field lines on the scale of the Larmor orbit. In fact, if the field is uniform and homogeneous, the perpendicular thermal flux is identically equal to zero since electrons cannot move across the field lines. In a tangled field, however, electrons can jump from one field line to another and thus conduct heat perpendicular to the field. Since we are dealing with a turbulent accretion flow with a tangled magnetic field, this is the regime of interest to us.

The physics of this regime of conduction has been discussed by Rechester & Rosenbluth (1978) and Chandran & Cowley (1998), who identified two important effects which are discussed in more detail in Appendix A.

First, since particles can move freely only along field lines, the characteristic effective mean free path is set by the correlation scale of the magnetic field l_B. In a hot accretion flow this scale is not known in general. However, it is likely that turbulent motions in the flow occur on a scale comparable to the local radius R, since this is the only characteristic scale in the problem. Very likely, the turbulent magnetic field will also have the same scale $l_B \sim R$. We parameterize this scale as $l_B = \xi R$. We expect $\xi \leq 1$ because turbulent fluctuations cannot have a scale larger than the local radius of the flow. We assume $\xi \sim 0.1$ throughout the paper.

Second, the magnetic field is inhomogeneous. Therefore, only a fraction $\vartheta < 1$ of the particles will be able to pass though the magnetic mirrors that will be present in the field, and it is only these particles that transport energy beyond a distance $\sim l_B$. For magnetic field strength fluctuations $\delta B \sim \langle B \rangle$, the fraction of free streaming particles is estimated to be $\vartheta \sim 0.3$.

Typically, hot accretion flows are highly collisionless, i.e., $\lambda \gg R \gtrsim l_B$. Therefore, we can write the thermal conduction coefficient as

$$\kappa_B \simeq n k_B v_T l_B \,\vartheta \simeq 10^{-2} n k_B v_T R \xi_{-1}\vartheta_{-1}, \tag{13}$$

where $\xi_{-1} = \xi/10^{-1}$ and $\vartheta_{-1} = \vartheta/10^{-1}$. Let us write the conductive heat flux in a form similar to that used for the viscous stress, namely

$$q_{\rm cond} = -\alpha_c \frac{c_s^2}{\Omega_K}\rho \frac{dc_s^2}{dx}, \tag{14}$$

where the dimensionless coefficient α_c is analogous to the Shakura-Sunyaev viscosity parameter α, and is given by

$$\alpha_c \simeq \frac{R}{H}\,\xi\vartheta\, F(e,p) \simeq 10^{-2}\xi_{-1}\vartheta_{-1}\, F(e,p). \tag{15}$$

Here we have used the fact that $v_T \simeq c_{se}$ and $H/R \sim c_s/v_{\rm ff} \sim c_s/\Omega_K R$, where H is the accretion disk scale height (in hot flows, $H \sim R$) and $v_{\rm ff}$ is the free-fall speed. The quantity $F(e,p)$ takes into account whether the conduction is dominated by protons or electrons; its numerical value is given in equation (A7).

In the presence of conduction the energy equation has an additional contribution from the divergence of $q_{\rm cond}$. The perturbed energy equation (3d) then becomes modified to

$$\frac{\rho_0}{\gamma-1}\frac{\partial a^2}{\partial t} - \rho_0 c_s^2 \frac{\partial \sigma}{\partial t} = (Q^+ - Q^-)' + \alpha_c \frac{\rho c_s^2}{\Omega_K}\frac{\partial^2 a^2}{\partial x^2}. \tag{16}$$

This equation together with equations (3a-c) yields the following modified dispersion relation:

$$\omega\left[\frac{\omega}{\gamma-1} + \frac{i(n-1)}{\tau_{\rm cool}} + \frac{ik^2R^2}{\tau_{\rm cond}}\right]\left(\omega^2 - \kappa_{\rm epi}^2 - k^2c_s^2\right) - \omega\left[\omega + \frac{i(2B/A-1)}{\tau_{\rm cool}}\right]k^2c_s^2 = 0, \tag{17}$$

where

$$\tau_{\rm cond} = \Omega_K R^2/\alpha_c c_s^2 \tag{18}$$

is the conductive time scale.

Figure 1 illustrates how thermal conduction modifies the thermal instability. The solid lines in the two panels show the real and imaginary parts of the roots of the dispersion relation (17) as functions of k. The parameters have the same values as before, namely $\kappa_{\rm epi}/\Omega_K = s = \sqrt{0.5}$, $c_s/\Omega_K R = 1$, $\tau_{\rm cool,ff}\Omega_K = 10$, $\gamma = 5/3$, $n = 1/2$. We have set the conductive time equal to $\tau_{\rm cool,ff}$ (for illustration), which corresponds to $\alpha_c \sim 0.3$.

Figure 1 shows that the imaginary parts of all three modes decrease rapidly with increasing k. For the particular parameters we have selected, the growth rate of the unstable thermal mode (curve 3) goes to zero at $k_{\rm crit}R \sim 1.5$, and the mode is stable for all $k > k_{\rm crit}$.

We may also analyze equation (17) analytically. In the large-k limit, the root corresponding to the thermal mode is equal to[3]

$$\omega = i\,\frac{(\gamma-1)}{\gamma}\left(\frac{(2-n-2B/A)}{\tau_{\rm cool}} - \frac{k^2R^2}{\tau_{\rm cond}}\right). \tag{19}$$

[3]In deriving this equation from (17) we have used the fact that the acoustic time-scale is, in general, shorter than the time-scale of the thermal mode, i.e., $\omega \ll kc_s$, and we have neglected $\kappa_{\rm epi}$ as before. In this case we can neglect ω^2 in the second brackets, so that equation (19) readily follows. It may seem that this procedure fails when the acoustic and thermal time-scales are comparable. This may happen when $kR \sim 1$ (for such perturbations, the sound crossing time is of order the dynamical time) and when $\tau_{\rm cool}$ and $\tau_{\rm cond}$ are also comparable to the dynamical time, which is $\sim \Omega_K^{-1}$. Nevertheless, even in this case, equation (19) works fairly well near the stability threshold. Indeed, at the threshold itelf, Im $\omega = 0$. Since further the thermal mode frequency has no real part, we have $\omega \sim 0$ near the threshold; so we may safely neglect ω^2 compared to $k^2c_s^2$ in the second brackets of equation (19).

Clearly, for large k, conduction stabilizes the thermal mode, for the reasons explained at the beginning of this section. Using the above relation, we can estimate the critical $k_{\rm crit}$ above which all k are stable:

$$k_{\rm crit}^2 R^2 = \frac{\tau_{\rm cond}}{\tau_{\rm cool}}\left(2 - n - 2\,\frac{B}{A}\right) = \frac{13}{6}\,\frac{\tau_{\rm cond}}{\tau_{\rm cool}}, \qquad (20)$$

where we have substituted $n = 1/2$ (free-free cooling) and $B/A = -1/3$ (Keplerian scaling).

We should comment here that in the theory described above, the conductivity κ_B is a quantity averaged over many field correlation lengths. Therefore, the results of the stability analysis are valid only for perturbations on scales much larger than l_B. (This is somewhat inconsistent since we have assumed that $l_B = \xi R \sim 0.1R$.) For small-scale perturbations with $k \gg (\xi R)^{-1}$, the local magnetic field is nearly homogeneous. Therefore, thermal conductivity is anisotropic; its perpendicular component is of order of κ_B (equation [A3]), while the conductivity along the field lines is much larger:

$$\frac{q_\|}{q_\perp} = \frac{C\rho c_s^3\,{\rm sgn}(\nabla T)}{-\alpha_c(\rho c_s^2/\Omega_K)(dc_s^2/dx)} \sim \frac{C\rho c_s^3}{\alpha_c \rho c_s^3 (H/R)} \sim \frac{5}{\alpha_c} \gg 1,$$

as follows from equations (14) and (12). Thermal instability along the magnetic field lines is then suppressed more strongly than in the analysis presented above.

4. THERMAL STABILITY OF THE HOT SETTLING FLOW SOLUTION

The hot settling flow solution (Medvedev & Narayan 2001) describes an optically thin two-temperature accretion flow onto a rotating neutron star. The flow extracts the rotational energy of the star and radiates it via free-free emission. The main properties of the solution are summarized in Appendix B.

Because the hot settling flow is cooling-dominated and radiates by free-free emission, it is intrinsically thermally unstable, as follows from equation (8b) for $\gamma > 1$, $n = 1/2$, and $B/A = -1/3$. However, thermal conductivity in the flow is enormous because of the very high temperature and large mean free path of particles. The coefficient of thermal conduction is estimated in Appendix B to be

$$\alpha_c \simeq 10^{-2}\xi_{-1}\vartheta_{-1}F(e,p), \qquad (21)$$

where $F(e,p)$ lies in the range $1 \le F(e,p) \lesssim 15$. Is this level of thermal conduction enough to stabilize the thermal mode?

The relevant stability criterion is given in equation (20). However, before we apply this criterion, we need to allow for the fact that, as shown in Appendix B, thermal conduction in the flow is so strong that it modifies even the equilibrium structure of the flow. In particular, the cooling time (6) is modified as per the substitution given in equation (B5) and becomes

$$\tau_{\rm cool} = \frac{4}{(9\alpha s^2 + 2\alpha_c)}\,\Omega_K^{-1} \simeq \frac{2}{\alpha_c}\,\Omega_K^{-1}, \qquad (22)$$

where we have assumed that $\alpha_c \gg \alpha s^2$, which is reasonable for typical parameters, e.g. $\alpha \sim 0.1$, $s \sim 0.1$, $\alpha_c \sim 0.1$. From equation (18) and using the self-similar solution (B1), we obtain the thermal conductive time[4]

$$\tau_{\rm cond} = \frac{3}{\alpha_c}\,\Omega_K^{-1}, \qquad (23)$$

Substituting $\tau_{\rm cool}$ and $\tau_{\rm cond}$ into the stability criterion (20), we then find

$$k_{\rm crit}R = \left[\frac{26\alpha_c}{9(9\alpha s^2 + 2\alpha_c)}\right]^{1/2} \simeq \sqrt{\frac{13}{9}} \simeq 1.2, \qquad (24)$$

that is, thermal modes with $kR \gtrsim 1$ are stable. This suggests that the hot settling flow with thermal conduction is marginally stable to the thermal instability.

Whether the mode $kR = 1$ itself is stable or not cannot be reliably determined from our local analysis. A global stability analysis is necessary to properly account for the effects of geometry and curvature, but this is beyond the scope of the present paper.

5. THERMAL STABILITY OF THE SLE SOLUTION

The SLE solution (Shapiro, Lightman, & Eardley 1976) was the first hot optically thin accretion solution discovered. This self-similar solution is best suited for accretion onto a black hole, though it could in principle be applied also for accretion onto a NS, by attaching a suitable boundary layer at its inner edge. The SLE solution as originally envisaged by Shapiro et al. (1976) was cooled by Compton-scattering of soft photons from an outer thin accretion disk. A local form of the solution (e.g., NY95b) involves local cooling via free-free emission. Some properties of this solution are discussed in Appendix C. The solution is thermally unstable (Piran 1978), as discussed in §2. The question we consider here is whether thermal conduction in the SLE solution is strong enough to eliminate the instability.

In Appendix C we show that the SLE flow is cooler than the hot settling flow and hence more collisional. Therefore, the collisionless thermal conductivity given by equation (A3) is not applicable. One should use instead equation (C6). To decide whether thermal conduction has any hope of stabilizing the mode, we determine an upper bound on the coefficient of thermal conduction. From equations (C6) and (C7) we find

$$\kappa_{SLE} \lesssim 1.1 \times 10^{20}\vartheta \dot m^{5/4}\alpha^{-5/2}r^{-15/8}. \qquad (25)$$

The cooling time then follows from (6) for a Keplerian flow ($s = 1$):

$$\tau_{\rm cool} = \frac{\rho c_s^2}{Q^-} = \frac{8}{9\alpha}\Omega_K^{-1}. \qquad (26)$$

Comparing equations (9) and (14), we write $\kappa = \alpha_c \rho c_s^2 k_B/(\Omega_K m_p)$. We then obtain a lower limit on the conductive time from equations (18) and (25):

$$\begin{aligned}\tau_{\rm cond} = \frac{\Omega_K R^2}{\alpha_c c_s^2} &= \left(\frac{k_B}{m_p}\frac{\rho c_s^2}{\kappa_{SLE}\Omega_K}\frac{R^2}{H^2}\right)\Omega_K^{-1} \\ &\gtrsim 13.3\vartheta^{-1}\dot m^{-1}\alpha^{-2}r^{1/2}\Omega_K^{-1}. \qquad (27)\end{aligned}$$

[4] Alternatively, we recall that in the hot settling flow $H/R \sim 1$ and $H/R \simeq c_s/v_{\rm ff} \simeq c_s/(\sqrt{2}\Omega_K R)$. Then $\tau_{\rm cond}$ readily follows from equation (18).

Here we have used the values of plasma parameters for a two-temperature zone (C2), which yields a shorter $\tau_{\rm cond}$ than for a one-temperature zone. We notice that the conduction time is significantly longer than the cooling time for typical parameters of the flow, e.g., $\dot{m} \sim 0.01$, $\alpha \sim 0.1$, $\vartheta \sim 0.1$, $r \lesssim 10^4$. Therefore, in contrast to the hot settling flow, thermal conduction in the SLE case is not strong enough to modify the equilibrium structure of the flow. We may thus directly apply the stability criterion (20). We then find that the critical $k_{\rm crit}$ below which the thermal mode is unstable is given by

$$k_{\rm crit}R \gtrsim 180\,(\vartheta_{-1}\dot{m}_{-2}\alpha_{-1})^{1/2}\, r_2^{1/4} \tag{28}$$

Since we find that $k_{\rm crit}R$ is very large, we conclude that the SLE solution is thermally unstable even in the presence of thermal conduction. This is in contrast to the hot settling flow where conduction has quite a dramatic effect. The reason for the difference is that the SLE solution is cooler and more collisional and therefore has a significantly lower coefficient of thermal conduction.

6. CONCLUSIONS

In this paper we have investigated how thermal conduction affects the stability properties of two hot, cooling-dominated, rotating accretion flows: the hot settling flow (Medvedev & Narayan 2001) and the SLE slim disk (Shapiro, Lightman, & Eardley 1976). Without thermal transport, both flows are thermally unstable since they balance viscous heating with free-free bremsstrahlung cooling (Piran 1978). Thermal conduction smears out temperature perturbations and can in principle suppress the instability. The stabilization effect is proportional to k^2R^2, where k is the wave-vector of a perturbation and R is the radius, and is thus strongest for small-scale perturbations, $kR \gg 1$.

To study the stability of the two rotating flows, we have used the shearing sheet approximation. We obtain the stability criterion (20), which gives the critical wave-vector, $k_{\rm crit}$, of a perturbation such that thermal modes with a larger spatial scale ($k < k_{\rm crit}$) are unstable. We claim that an accretion flow is thermally stable if $k_{\rm crit}R \lesssim 1$ and unstable if $k_{\rm crit}R \gg 1$. Technically, for perturbations with $k \sim 1/R$, the effects of geometry and curvature must be properly taken into account. This requires a global analysis, which we do not attempt.

Because of the very high (nearly virial) temperature of the gas in the hot settling flow, the particle mean-free-path is much larger than the temperature gradient scale (which is comparable to R). In this regime, the Spitzer theory of thermal conduction cannot be applied. Instead, one should consider collisionless transport theory including the effects of tangled (but dynamically unimportant) magnetic fields. The thermal flux is then given by equation (14). We find that this flux is energetically very important in the hot settling flow and modifies even the equilibrium flow (see Appendix B). We allow for the modification and then analyse the thermal stability of the resulting flow. We obtain the stability criterion (24), which indicates that modes with $kR \gtrsim 1$ are stable. This suggests that the hot settling flow is thermally stable in the presence of thermal conduction.

The SLE solution is cooler than the hot settling flow, and particle collisions are more frequent. The thermal flux in this flow is described by collisional transport theory suitably modified for the presence of tangled magnetic fields. We find that thermal transport in the SLE flow is not very important and that the flow is thermally unstable even when conduction is included (see eq 28).

We have thus answered the question with which we began the paper (§1). However, in the process we have come up with an unexpected realization: conduction in hot dilute accretion flows may be so large as to substantially modify the equilibrium flow. This raises three interesting issues.

First, in the basic hot settling flow without conduction, it has been shown that the energy source for viscous heating of the gas is ultimately derived from the rotation of the central star (Medvedev & Narayan 2001). If, as we have shown here, conductive heating is more important than viscous heating, then one could ask where the energy for the conductive flux comes from. Clearly, the energy must be derived from accretion. A complete global solution, including conduction, should be able to trace the flow of energy from the hot inner regions of the accretion flow, where gravitational energy is released, to the hot outer regions, where the gas is approximately described by the self-similar settling flow solution.

Second, if particle transport can have such a strong influence on energy balance, what about angular momentum balance? Particles that move from one radius to another will certainly have some effect on the angular momentum of the gas, but the exact details are uncertain when particle trajectories are constrained by the magnetic field. For simplicity, we have ignored in this paper angular momentum transport by the collisionless particles. This effect should be included for consistency when the nature of the interactions is understood.

Third, apart from the hot settling flow, two other hot accretion flows with nearly virial temperature are known: the ADAF and the CDAF. One suspects that energy conduction (and angular momentum transport) by collisionless particles might be important for these solutions as well, and might modify their equilibrium structures. We note that a CDAF is very different from an ADAF because convection has a strong effect and changes the radial structure of the flow drastically (Narayan et al. 2000; Quataert & Gruzinov 2000). If we now add another form of transport, there could well be additional effects.

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APPENDIX

COLLISIONLESS THERMAL CONDUCTION IN TANGLED MAGNETIC FIELDS

In this Appendix we discuss the essence of the thermal transport theory in collisionless systems threaded by weak tangled magnetic fields.

In the collisionless regime, the mean-free-path λ of an electron is comparable to or larger than the temperature gradient scale $\lambda \gtrsim T_e/|\nabla T_e|$. The heat flux in this case is $\sim \rho c_s^3$. In a collisionless magnetized plasma, electrons stream freely along the field lines only. In the transverse direction, they are tied to the field lines on the Larmor scale. In a tangled field, however, electrons can jump from one field line to another and thus conduct heat across the field. This is the regime of interest to us. The physics of this regime of conduction has been discussed by Rechester & Rosenbluth (1978) and Chandran & Cowley (1998).

Let us consider a tangled magnetic field with a characteristic correlation scale l_B. We assume that the electron Larmor radius ρ_e in this field is much smaller than the mean free path λ. If l is the path length measured along a field line, the separation between two closely neighboring field lines grows as

$$d(l) \sim d(0)e^{l/L_K}, \tag{A1}$$

where $d(0)$ is their initial separation and L_K is the Kolmogorov-Lyapunov length which, in general, depends on the field spectrum. Since there is only one characteristic scale in the problem, l_B, we expect $L_K \sim l_B$. Let us assume that at an initial instant, an electron drifted a distance $d(0) \sim \rho_e$ from its initial field line in the perpendicular direction. Once it starts moving along a new field line, its new path diverges from its initial path according to equation (A1). After the particle travels the distance

$$L_{RR} \sim l_B \ln(l_B/\rho_e) \tag{A2}$$

measured along the field line, the separation becomes of order l_B. Any subsequent motion of the electron then becomes uncorrelated with its initial field line. The distance L_{RR} is called the Rechester-Rosenbluth length.

If v_T is the characteristic electron thermal speed, then from dimensional considerations it follows that the thermal flux is described by equation (9) with an effective conductivity $\kappa_B \sim nk_Bv_Tl_B$. A more rigorous analysis (Chandran & Cowley 1998) shows that for large-scale perturbations, $kl_B \lesssim 1$, one can take the ensemble average. Then the thermal conductivity becomes isotropic (assuming isotropic magnetic turbulence) and reads as

$$\kappa_B \simeq nk_Bv_Tl_B\,\vartheta\,\frac{\min(\lambda, L_{RR})}{L_{RR}}, \tag{A3}$$

where the function $\min(a,b)$ denotes the smaller of a and b. The quantity $\vartheta \leq 1$ takes into account the fact that only a fraction of particles (those which are not trapped by magnetic mirrors) can move far enough to transport heat; the rest will remain trapped in their local magnetic wells and be unable to transport energy beyond a distance $\sim l_B$. We calculate ϑ below.

There are three substantially different regimes of thermal conduction. First, in the collisionless limit, $\lambda \gg L_{RR}$, we have $\kappa_B \sim nk_Bv_Tl_B\vartheta$, i.e., l_B plays the role of an effective mean-free-path and only non-trapped particles contribute. Second, in the semi-collisionless limit, $\rho_e \ll \lambda < L_{RR}$, we have $\kappa_B \sim \kappa_{\rm Sp}\vartheta l_B/L_{RR}$. That is, the standard Spitzer collisional transport is affected by the presence of tangled fields: the mean-free-path is set by collisions but most of the particles are still trapped in magnetic mirrors. Third, in the strongly collisional limit: $\lambda \ll \rho_e$, the particles do not move along the field lines, magnetic mirroring is inefficient and, thus, κ_B approaches the classical Spitzer value. Note also that in the first and second cases, the heat flux is anisotropic for small-scale perturbations, $kl_B \gg 1$, because the magnetic field is locally ordered. The flux is given by equation (9) with $\kappa = \kappa_B$ across the field lines, and by equation (12) or equation (9) with $\kappa = \kappa_{\rm Sp}$, depending on plasma collisionality, along the field lines.

Now we estimate the fraction of particles, ϑ, which can pass through magnetic mirrors. Let us consider a particle which moves along a field line from a region "1" with a weak field to a region "2" with a stronger field. If the field gradient scale is much larger than the Larmor radius (which is the case for most astrophysical systems), the particle's magnetic moment $\mu = mv_\perp^2/2B$ ($v_\perp$ is the velocity component perpendicular to the local field) is conserved as an adiabatic invariant. That is $mv_{\perp 1}^2/2B_1 = mv_{\perp 2}^2/2B_2$. The energy of the particle is also constant. At the point where the particle is reflected its parallel velocity vanishes, therefore $v_{\perp 2}^2 = v_{\perp 1}^2 + v_{\parallel 1}^2$. Combining these two equations we obtain that particles are reflected from the magnetic mirror if their pitch angles are greater than the minimal pitch angle, defined as:

$$\sin^2\theta_m = B_{\rm min}/B_{\rm max}. \tag{A4}$$

The particles with $\theta < \theta_m$, which can pass though the mirror, form a "loss cone", the solid angle of which is

$$\Omega_{lc} = 2\pi(1 - \cos\theta_m).$$

There is a second loss cone in the opposite direction along the field line, due to symmetry. Assuming an isotropic distribution of particle velocities (in the weak field region), we estimate the fraction of free streaming particles as

$$\vartheta = 2\Omega_{lc}/4\pi = 1 - (1 - B_{\rm min}/B_{\rm max})^{1/2} \sim 0.3, \tag{A5}$$

where we assumed that the amplitude of magnetic turbulent fluctuations in an accretion flow is typically $\delta B \sim \langle B\rangle$, i.e., $B_{\rm max} \sim 2B_{\rm min}$.

In hot accretion flows, the Coulomb mean-free-path is often much larger than the size of the flow, so that $\lambda \gtrsim L_{RR}$. It is convenient then to write the conductive heat flux as in equation (14),

$$q_{\rm cond} = -\alpha_c \frac{\rho c_s^2}{\Omega_K}\frac{dc_s^2}{dx}, \qquad \text{where} \qquad \alpha_c = \sum_{i=e,p} \kappa_B \frac{m_i \Omega_K}{k_B \rho c_s^2}\frac{dc_{si}^2/dx}{dc_s^2/dx} \simeq 2^{1/2}\frac{l_B}{H}\vartheta F(e,p) \tag{A6}$$

is the coefficient of thermal conduction, $i = e, p$ because both species contribute to heat transport, and $F(e,p) \equiv \max\left[F(e), F(p)\right]$ properly corrects for the two-temperature flow, and where $H/R \simeq c_s/v_{\rm ff} = c_s/(\sqrt{2}\Omega_K R)$ is of order unity in hot flows, H is the accretion disk scale height and $v_{\rm ff}$ is the free-fall speed. We also used here that $v_{Te,p} \simeq c_{se,p}$. Thermal conduction is dominated by electrons in a one-temperature flow because they are lighter than protons. In a two-temperature flow, however, the protons may be much hotter than the electrons and, hence, will dominate the conduction. We write the relative contributions of electrons and protons to thermal transport in two- and one-temperature zones:

$$\begin{aligned} F(e)_{2T} &= \frac{m_e}{m_p}\frac{c_{se}}{c_s}\frac{dc_{se}^2/dx}{dc_s^2/dx}, \qquad & F(e)_{1T} &= \frac{1}{2\sqrt{2}}\sqrt{\frac{m_e}{m_p}} \simeq 15.2, \\ F(p)_{2T} &= 1, \qquad & F(p)_{1T} &= \frac{1}{2\sqrt{2}} \simeq 0.35, \end{aligned} \tag{A7}$$

where $c_s^2 = c_{sp}^2 + (m_e/m_p)c_{se}^2 = nk_B(T_p + T_e)$. We have made use of the fact that in a two-temperature zone ($T_p \gg T_e$) $c_s^2 \simeq c_{sp}^2$ and in a one-temperature zone ($T_p = T_e$) $c_s^2 \simeq 2c_{sp}^2$.

PROPERTIES OF THE HOT SETTLING FLOW ONTO A NEUTRON STAR

The first analytical solution for a hot accretion flow settling onto a rapidly rotating NS was found by Medvedev & Narayan (2001). We briefly present here some relevant results and derive some new results. The hot settling flow is geometrically thick with the vertical thickness being comparable to the local radius of the flow, $H \sim R$. The flow is hot with the protons being at nearly the virial temperature. The flow rotates with a sub-Keplerian angular velocity. The flow is optically thin and is cooled by free-free emission. The flow is powered by the rotational energy of the central star; hence the flow parameters (all but the radial velocity) are independent of the mass accretion rate. As in the case of an ADAF, the mass accretion rate must not exceed a critical value which is of order a few percent of the Eddington value; otherwise, the cooling is too strong and a thin Shakura-Sunyaev disk forms.

The settling flow consists of two zones: an inner two-temperature zone for $r \lesssim 10^{2.5}$ ($r = R/R_S$, where $R_S = 2.95 \times 10^5 m$ cm is the Schwarzchild radius), and an outer one-temperature zone for $r \gtrsim 10^{2.5}$. Each zone is described by a self-similar solution. The two- and one-temperature solutions read:

$$\begin{aligned} \rho_{2T} &\simeq 8.9\times10^{-3} m^{-1}\alpha s^2 r^{-2} {\rm g/cm^3}, \qquad & \rho_{1T} &\simeq 4.9\times10^{-2} m^{-1}\alpha s^2 r^{-2}\ {\rm g/cm^3}, \\ \theta_{p,2T} &= (1/6) r^{-1}, & \theta_{p,1T} &= \theta_{p,2T}/2, \\ \theta_{e,2T} &\simeq 11 r^{-1/2}, & \theta_{e,1T} &\simeq 153 r^{-1}, \\ \Omega_{2T} &\simeq 7.2\times10^4 m^{-1} s r^{-3/2}\ {\rm rad/s}, & \Omega_{1T} &= \Omega_{2T}, \\ v_{2T} &\simeq 1.4\times10^8 \dot m \alpha^{-1} s^{-2} r^0\ {\rm cm/s}, & v_{1T} &\simeq 2.6\times10^7 \dot m \alpha^{-1} s^{-2} r^0\ {\rm cm/s}, \end{aligned} \tag{B1}$$

where $\theta_p = k_B T_p/m_p c^2$ and similarly for the electrons, $m = M_{NS}/M_\odot$, $\dot m = \dot M/\dot M_{\rm Edd}$ is the mass accretion rate in Eddington units, $\dot M_{\rm Edd} = 1.4\times10^{18} m$ g/s, α is the viscosity parameter, and $s = \Omega_{NS}/\Omega_K(R_{NS})$ is the NS spin in units of the Keplerian angular velocity at the NS radius. Note that, except for v, none of the quantities depend on the accretion rate $\dot m$. The one-temperature self-similar solution is valid from $r \sim 10^{2.5}$ to approximately

$$r_{ss} \sim \left(74\alpha^2 s^2/\dot m\right)^2 \sim 5.5\times10^3 \alpha_{-1}^4 s_{-1}^4 \dot m_{-4}^{-2}. \tag{B2}$$

Beyond this radius the solution is not self-similar.

The above self-similar solutions were obtained without taking into account thermal conductivity of the gas. As we will now see, thermal conduction in a hot settling accretion flow is very large, so that the thermal flux is energetically important and modifies the structure of the equlibrium settling flow itself.

To be consistent with our simple analysis of the thermal instability in §2, it is sufficient to consider the one-temperature regime only. The hot settling solution is obtained from the condition that the total local heating rate is equal to the total cooling rate. Including the contribution from the divergence of the conductive flux, the condition now reads

$$\vec\nabla\cdot\vec q_{\rm cond} \equiv -\frac{1}{R^2}\frac{\partial}{\partial R}R^2\frac{\alpha_c\rho c_s^2}{\Omega_K}\frac{\partial c_s^2}{\partial R} = Q^+ + Q^-, \tag{B3}$$

where $q_{\rm cond}$, Q^-, and Q^+ are given by equations (14), (4). Since the additional heat flux term has exactly the same radial dependence as the other two terms, the power-law scalings given by equations (B1) remain valid, but with their pre-factors changed. Let us write the new solution as $\rho = \hat{\rho} r^{-2}$, $\theta_p = \hat{\theta}_p r^{-1}$, etc.. With these definitions, the energy balance equation (B3) takes the form:

$$\frac{9}{4}\hat{\rho} c^2 \hat{\theta}_p \hat{\Omega}_K r^{-9/2} \left(\alpha s^2 + \frac{4}{3}\hat{\theta}_p \alpha_c\right) = \mathcal{C}\hat{\rho}^2 c \hat{\theta}_p^{1/2} r^{-9/2}. \tag{B4}$$

Clearly the effect of thermal conduction is to to re-define the quantity αs^2 to

$$\alpha s^2 \to \alpha s^2 + \frac{2}{9}\alpha_c, \tag{B5}$$

where we have used $\hat{\theta}_p = 1/6$. With this re-definition, we may continue to use equations (B1) for the equilibrium flow.

Next, we estimate the typical value of the thermal conduction coefficient in the hot settling flow. From equations (15) and (A7) we have

$$\alpha_c = 2^{-1/2}\frac{R}{H}\,\xi\vartheta F(e,p) \simeq 10^{-2}\xi_{-1}\vartheta_{-1}F(e,p). \tag{B6}$$

From (A7) and (B1) we obtain $F(e)_{2T} \simeq 0.15\, r^{3/4} \lesssim 10.9$ because the two-temperature zone exists for $r \lesssim 10^{2.5}$. That is, in the two-temperature zone $F(e,p)$ changes from ~ 1 in the inner parts to ~ 10 in the outer parts and then smoothly approaches the value of ~ 15 in the one-temperature zone. Thus, the overall quantity $F(e,p) \equiv \max[F(e), F(p)]$ lies in the range $1 \leq F(e,p) \lesssim 15$ for the entire flow. Moreover, we note that $F(e) \lesssim F(p)$ only for small radii $r \lesssim 13$, where the self-similar settling solution is not terribly accurate because of Comptonization. Therefore, we see that thermal conduction is dominated by the electrons over nearly the entire accretion flow.

Finally, we demonstrate that (13) is the relevant limit of equation (A3) and, hence, that the above estimate for the thermoconduction coefficient is accurate. For this, we must demonstrate that $\lambda \gtrsim L_{RR}$ in the settling accretion flow (see equation A3).

From the estimates above we have $\alpha s^2 + (2/9)\alpha_c \simeq 2.2 \times 10^{-3}\xi_{-1}\vartheta_{-1}F(e,p)$. From equation (11), the electron mean-free-path, $\lambda \sim 0.57\theta_e^2/\rho$, normalized by the local radius of the flow, is estimated in both zones to be

$$\lambda_{2T}/R \simeq 12\left[\xi_{-1}\vartheta_{-1}F(e,p)\right]^{-1}, \qquad \lambda_{1T}/R \simeq 17\left[\xi_{-1}\vartheta_{-1}F(e,p)\, r_3\right]^{-1}. \tag{B7}$$

The proton mean free path is comparable to or larger than the above estimate. To calculate L_{RR} the Larmor radius of the electrons is needed. It may be estimated as follows. The magnetic pressure $P_m = P_{\rm gas}/\beta$ yields

$$B \simeq \left(8\pi\rho c_s^2/\beta\right)^{1/2}.$$

Since $c_s^2 = c_{sp}^2 + (m_e/m_p)c_{se}^2 \sim c_{sp}^2$ to within a factor of two and the thermal velosity of particles is $v_{Te} \simeq c_{se}$, we obtain

$$\rho_L \simeq v_{Te}/\Omega_B \simeq 1.1 \times 10^{-8}\left(\frac{\beta\theta_e}{\rho\theta_p}\right)^{1/2}, \tag{B8}$$

where $\Omega_B = eB/m_e c$ is the electron cycloron frequency. For a typical $\beta \simeq 10$ (see e.g., M00,QG99), the normalized electron Larmor radii in the two- and one-temperature zones are:

$$\rho_{L,2T}/R \simeq 2.2 \times 10^{-10} m^{-1/2} r^{1/4}\left[\xi_{-1}\vartheta_{-1}F(e,p)\right]^{-1/2}, \qquad \rho_{L,1T}/R \simeq 5.1 \times 10^{-10} m^{-1/2}\left[\xi_{-1}\vartheta_{-1}F(e,p)\right]^{-1/2}. \tag{B9}$$

Since the two quantities differ at most by a factor of order unity, we use $\rho_L/R \sim 5 \times 10^{-10}$ as a representative value. Now we can estimate the Rechester-Rosenbluth length from equation (A2),

$$L_{RR}/R \simeq \xi \ln\left[\xi/(\rho_L/R)\right] \simeq 10^{-1}\xi_{-1}(19 + \ln \xi_{-1}) \sim 1.9\xi_{-1}. \tag{B10}$$

Finally, we have

$$\lambda_{2T}/L_{RR} \simeq 6.3\left[\xi_{-1}^2\vartheta_{-1}F(e,p)\right]^{-1}, \qquad \lambda_{1T}/L_{RR} \simeq 8.9\left[\xi_{-1}^2\vartheta_{-1}F(e,p)\right]^{-1} \tag{B11}$$

We see that in the two-temperature zone, the mean free path is large, $\lambda \gtrsim L_{RR}$, so that equations (13) and (A3) are justified. In the one-temperature zone, the result is marginal: $\lambda \sim L_{RR}$. However, considering that (i) the transition between the regimes with $\lambda \ll L_{RR}$ and $\lambda \gg L_{RR}$ is not sharp in reality, (ii) the "boundary" itself, $\lambda \sim L_{RR}$, is uncertain to within a numerical factor of a few, and (iii) the one-temperature zone is rather small [c.f., equation (B2)], we believe that we may use equation (13) throughout the flow. The error will not be greater than a factor of order unity.

PROPERTIES OF THE SLE SOLUTION

The SLE slim disk solution was the first hot accretion solution found. The SLE solution is optically thin. It is cooling-dominated via free-free emission (the original paper by SLE considered Compton cooling). As in other hot solutions, there are both two- and one-temperature zones in this flow. In the two-temperature zone, the normalized electron and proton temperatures are related as follows:

$$\theta_e = \left(\frac{\pi \ln \Lambda}{12\alpha_f}\right)^{1/2} \theta_p^{1/2} \simeq 23\theta_p^{1/2}, \tag{C1}$$

where α_f is the fine structure constant and $\ln \Lambda = 15$. This equation is easily obtained from the equality of the electron cooling rate via Bremsstrahlung and the energy transfer rate from the protons to the electrons via Coulomb collisions, $Q^- = Q_{\rm Coul}$; the expressions for these rates are given in Medvedev & Narayan (2001). In the one-temperature zone, $T_p = T_e$, i.e., $\theta_e = (m_p/m_e)\theta_p$. Following Shapiro, Lightman, & Eardley (1976), we obtain the following solutions:

$$\begin{array}{rclrcl} c_{s,2T} & \simeq & 6.5\times10^9 \dot m^{1/4}\alpha^{-1/2}r^{-3/8}\ {\rm cm/s}, & c_{s,1T} & \simeq & 1.9\times10^9 \dot m^{1/4}\alpha^{-1/2}r^{-3/8}\ {\rm cm/s}, \\ \rho_{2T} & \simeq & 2.9\times10^{-3} m^{-1}\dot m^{1/4}\alpha^{1/2}r^{-15/8}\ {\rm g/cm}^3, & \rho_{1T} & \simeq & 1.2\times10^{-1} m^{-1}\dot m^{1/4}\alpha^{1/2}r^{-15/8}\ {\rm g/cm}^3, \\ (H/R)_{2T} & \simeq & 3.1\times10^{-1}\dot m^{1/4}\alpha^{-1/2}r^{1/8}, & (H/R)_{1T} & \simeq & 2.2\times10^{-1}\dot m^{1/4}\alpha^{-1/2}r^{1/8}, \end{array} \tag{C2}$$

From the above equations, the dimensionless temperature in the two-temperature zone ($r \lesssim 10^2$) is $\theta_p = (c_s/c)^2 \simeq 0.05\ r^{-3/4}$ which is significantly smaller than the temperature of the hot settling flow in which $\theta_p \simeq 0.2r^{-1}$. In the one-temperature zone, $theta_p \simeq 0.004\ r^{-3/4}$ is again smaller that the temperature of the hot settling flow, $\theta_p \simeq 0.1\ r^{-1}$, up to a very large radius $r \sim 4\times10^6$. We see that the temperature of the gas in the SLE disk is smaller than in the hot settling flow. Therefore, we expect that collisions are more frequent and they may change the thermal properties of the gas. Indeed, we denonstrate now that $\lambda \lesssim L_{RR}$ in the SLE disk, that is, thermal conduction is in the semi-collisionless regime (see Appendix A).

From equations (11) with $T = c_s^2 m_p/k_B$ and (C1), we obtain the normalized collisional mean-free-paths

$$\lambda_{2T,p}/R \simeq 2.9\dot m_{-2}^{3/4}\alpha_{-1}^{-5/2}r_2^{-5/8}, \qquad \lambda_{2T,e}/R \simeq 0.3\dot m_{-2}^{1/4}\alpha_{-1}^{-3/2}r_2^{1/8}, \qquad \lambda_{1T}/R \simeq 2.2\times10^{-2}\dot m_{-2}^{3/4}\alpha_{-1}^{-5/2}r_2^{-5/8}. \tag{C3}$$

The normalized electron Larmor radius follows from equation (B8):

$$\rho_{L,2T}/R \simeq 1.3\times10^{-10} m^{-1/2}\dot m_{-2}^{-1/4}r_2^{1/8}, \qquad \rho_{L,1T}/R \simeq 3.6\times10^{-11} m^{-1/2}\dot m_{-2}^{-1/8}\alpha_{-1}^{-1/4}r_2^{-1/16}, \tag{C4}$$

where we take $r \sim 100$ as a characteristic radius at which a one-temperature flow becomes two-temperature. Since $\rho_L \propto (Tm)^{1/2}$, the proton Larmor radius is $\sqrt{m_p/m_e} \sim 43$ times larger. We estimate L_{RR} from equation (A2) as

$$L_{RR}/R \sim -\xi \ln(\rho_L/R) \sim 2.5\xi_{-1} \tag{C5}$$

for the electrons and it is slightly less for the protons. Comparing this with (C3), we see that $\lambda \ll L_{RR}$ in that part of the disk where the electron thermo-conduction dominates. Only in the inner parts of the disk, where the proton conductivity becomes large, do we have $\lambda \gtrsim L_{RR}$.

Finally, we calculate the thermal conductivity of the gas in the SLE disk. It is determined by equation (A3) with $\lambda < L_{RR}$:

$$\kappa_B = \kappa_{\rm Sp}\vartheta(l_B/L_{RR}) = \kappa_{\rm Sp}\vartheta/|\ln(\rho_L/R)| \simeq 4\times10^{-3}\vartheta_{-1}\kappa_{\rm Sp} \ll \kappa_{\rm Sp}. \tag{C6}$$

This thermoconductivity is about two orders of magnitude smaller than the Spitzer value. As we discussed above, the protons may significantly contribute to the thermal conduction flux and may even dominate the electron contribution in the inner parts of the disk. The proton contribution, which is $\kappa_{\rm Spp} = \sqrt{m_e/m_p}\kappa_{\rm Spe}(T_p/T_e)^{5/2}$ (see equations [10], [11]), becomes dominant in the two-temperature zone with $T_p \gg T_e$. In one-temperature zone, the electron contribution dominates, since $\kappa_{\rm Spp} \simeq (1/43)\kappa_{\rm Spe}$. Using the self-similar SLE solution (C2), we calculate the *nominal* Spitzer thermal conductivity in the SLE disk

$$\kappa_{{\rm Sp},2T} \simeq 2.7\times10^{21}\dot m^{5/4}\alpha^{-5/2}r^{-15/8}, \qquad \kappa_{{\rm Sp},1T} \simeq 2.5\times10^{20}\dot m^{5/4}\alpha^{-5/2}r^{-15/8} \tag{C7}$$

in CGS units. The real value of κ is less than the Spitzer value by the factor given in (C6).

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FIG. 1.— Real part (left panel) and imaginary part (right panel) of the frequencies of three modes. The curves labeled 1 and 2 refer to the two acoustic modes, and the curves labeled 3 refer to the thermal mode. The dashed curves correspond to the dispersion relation (5), which does not include thermal conduction. The following parameter values were used: $\kappa_{\rm epi}/\Omega_K = s = \sqrt{0.5}$, $\tau_{\rm cool,ff} = 10\Omega_K^{-1}$, $c_s = \Omega_K R$, $\gamma = 5/3$. The mode frequencies are normalized by the Keplerian frequency. The solid curves correspond to the dispersion relation (17), which includes thermal conduction. Here, $\tau_{\rm cond} = \tau_{\rm cool,ff}$, and the other parameters are the same as before.